

PROPOSED CLASSIFICATION OF MANGROVE SPECIES BASED ON LEAF FRACTAL DIMENSIONS

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ABSTRACT

The study utilized the fractal dimensions of the leaves of mangrove species locally found in the City of Tanguib to determine if the same can be used for classification purposes. The species used were *Rhizophora mucronata*, *Avicennia marina*, *Pemphis acidula*, *Sonneratia alba*, and *Acrostichum aureum*. Findings revealed that fractal dimensions could be used to differentiate one mangrove species from another through their leaf roughness ($f = 14.400$, $p = 0.000$). However, leaf fractal dimensions alone could not differentiate the following mangrove species within groups (*Rhizophora mucronata* and *Avicennia marina*); (*Acrostichum aureum* and *Pemphis acidula*); (*Sonneratia alba*). In effect, leaf fractal dimensions identified only the following groupings: (*Rhizophora mucronata*, *Avicennia marina*); (*Acrostichum aureum*, *Pemphis acidula*) and (*Sonneratia alba*). The results might be due to the small sample sizes used for some of the mangrove species. In particular, the standard deviations of the fractal dimensions of the *Sonneratia alba* and *Avicennia marina* might have been over-estimated because of the small sample sizes. The empirical probability of misclassification using the technique was approximately 4.5%.

Keywords: leaf fractal dimensions, taxonomy, mangrove species

1.0 Introduction

Central to research ranging from physiology to conservation biology is the problem of sorting individuals into species categories. The importance of accurate species descriptions or accurate classification stems from their impact on estimations of species' habitat ranges, physiologic tolerance, and population sizes. Knowlton and Jackson (1994) noted, in fact, that these errors have consequences for our understanding of ecologic and evolutionary theory as well as the management and mitigation of the effects of global climate change. Establishing techniques that provide accurate characterization of species and that minimizes ambiguity in allocating

individuals to species groups is important. Carlo et al. (2011), however, averred that it is important to realize that traits used in discerning the evolutionary status of groups are not necessarily practical tools for identification. Moreover, traits that can be used to identify specimens do not necessarily provide information about the evolutionary relatedness of species (Carlo et al., 2011). This study proposes the use of fractal dimensions as bases for taxonomic classification of mangrove species because of the sensitivity of fractal measures in detecting very small changes in the morphological features of plants and animals. Classification methods

based on shape and morphological forms have been done in the past using Elliptic Fourier Analysis (EFA) and Geometric Morphologic Analysis (GMA) (Neto et al., 2005).

Rodriguez (2013), for instance, found that the fractal dimensions of fruit tree leaves endemic in the Philippines are significantly different across fruit trees but highly similar for the same fruit trees. These findings imply that the fractal dimensions of the leaves of fruit trees can be used for classification purposes. Palmer (1992) also found the same phenomenon for the fractal dimensions of leaves of trees endemic in the United States. In general, fractal dimensions have been used successfully for classification and identification purposes in other various fields. Barrera (2013) used fractal dimensions for handwriting analysis; Krummel (1987)

applied fractal on soil fertility analysis; Selvam (2007) described the fractal dimensions of puffer fish; Manchiatto et al., (2004) used fractal analysis for the seismic activities in Italy.

The use of fractals in classification and identification of mangrove species in the Philippines has not been tried out. This study is a pioneering investigation in this direction.

2.1 Concept of a Fractal and Fractal Dimensions

Classical geometry considers objects that have integral dimensions: points have zero dimension, lines have one dimension, planes have two dimensions and cubes have three dimensions. Within a plane, one can represent points and straight lines and other geometric objects as shown in figure 1:



Figure 1. A fractal object in a plane

It is possible to represent geometric objects within a plane that are neither points nor lines like the squiggly line above. This squiggly geometric object cannot have dimension equal to 1 because it fills up more space than a line; it cannot have dimension equal to 2 because it does not form an area. Hence, its dimension λ has to be between 1 and 2 like $\lambda = 1.63$. We will say that the squiggly

line is a fractal (a geometric object having fractional dimension). The fractal dimension of an object defines its roughness, ruggedness or fragmentation. The higher the fractal dimension, the more rugged and irregular looking is the object. Thus, although fractals are rough and irregular objects, the pattern of irregularities are repeated over and over again. This is called the self-similarity

property of fractal. Benoit Mandelbrot (1967) is acknowledged as the mathematician who opened roughness as a legitimate topic for investigation in modern science. He claimed that nature and natural processes are fractals, while uniform, smooth and continuous patterns are man-made concepts and pervade mathematical analysis. He also said that by introducing “randomness” into the situation, one gets more realistic fractal representations.

After the publication of Mandelbrot’s book: *Fractals: The Geometry of Nature*, many scientists used fractals with great success (Cohen, 1987 on fractal antennae; Krummel et al., 1987) on forest fractals and others. It has found applications in various disciplines as well as in many areas of practical technology.

In Padua (2012), fractal geometry was translated to statistical language. A probability distribution akin to Pareto’s distribution for incomes was proposed as a model for fractal random variables X:

$$(1) \dots f(x) = \frac{(\lambda-1)}{\theta} \left(\frac{x}{\theta}\right)^{-\lambda}, x \geq \theta, \lambda > 0$$

where λ = fractal dimension of x , $\theta = \inf\{x\}$. A maximum – likelihood estimator for λ based on a random sample of size n was provided as:

$$(2) \dots \hat{\lambda} = 1 + n \left(\sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right)\right)^{-1}$$

He then proceeded to show that for n=1:

$$(3) \dots z = \hat{\lambda} \log\left(\frac{x}{\theta}\right) - 1 \underset{\sim}{d} \text{Exp}(\lambda - 1) \text{ or:}$$

$$(4) \dots q(z) = (\lambda - 1) \exp[-(\lambda - 1)z]$$

For a random sample of size n, the random variable:

$$(5) \dots q = \hat{\lambda} \sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right) - n$$

Has the same distribution as $q^s = \sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right) = \sum_{i=1}^n Z_i$. The distribution of (5) is therefore *Gamma* $\left(n, \beta = \frac{1}{\lambda-1}\right)$ where, $\lambda > 1$.

$$(6) \dots h(q) = \frac{(\lambda-1)^n}{\Gamma(n)} q^{n-1} e^{-q(\lambda-1)}, q > 0, \lambda > 1.$$

$$h(q) = \frac{(\lambda-1)^n}{\Gamma(n)!} q^{n-1} e^{-q(\lambda-1)}$$

Thus, if we have one sample of a species and if we are able to estimate its (geometric) fractal (see for example some available freeware like FRAK.OUT), then we are able to compare the fractal dimension for species (say, λ_1) with the specimen (λ_2):

$$(7) \dots \mu = |\lambda_1 - \lambda_2|.$$

We approximate the distribution of μ by an exponential distribution and obtain:

$$(8) \dots \delta_s = P(u \geq \varepsilon) = \frac{1}{2} \left(1 + \exp(-\varepsilon^{(\lambda_2-1)})\right),$$

a similarity index where λ_2 = fractal dimension q specimen species. We refer to (8) as a similarity index. As the difference $\varepsilon = |\lambda_1 - \lambda_2|$ increases, the similarity index decreases. If $\lambda_1 - \lambda_2$ (hence, $\varepsilon = 0$), the fractal dimensions are identical and the two documents are 100% similar. This means that the two species contains exactly the same fractal characteristics: straight lines, curves, strokes, spacings, slants and so on, and, must therefore belong to the same species.

It is also possible to determine what values of ε will yield high similarity index thus:

$$(9) \dots \delta_s \geq 1 - \alpha \Leftrightarrow \varepsilon \leq \left[\log\frac{1}{1-2\alpha}\right] \left[\frac{1}{\lambda_2-1}\right]$$

For instance, if $a = 0.05$, then the values of ϵ above will indicate 95% similarity index or greater.

3.0 Research Design and Methods

The study is designed to assess the viability of using fractal analysis in classifying and identifying the taxonomic classification of mangrove species based on the fractal dimensions of their leaves. For each species of mangrove, we randomly collected at least five (5) leaves. The leaves were carefully washed to ensure that only the leaf shape, form and structures are photographed. Their fractal dimensions were calculated using the available FRAKOUT software. A digital camera was mounted and fixed approximately one ft. (12 inches) from a platform where the specimen is mounted on a piece of white, 11" x 8", cardboard. Room lighting and dust contamination were controlled in order to ensure that

only the specimen's features are caught on camera.

The percent correct identification is then calculated as:

$$(10) \dots \text{Percent Correct Identification (PCI)} \text{ per species} = \frac{\text{no. of } \delta_s \text{ greater or equal to } 99\%}{9} \times 100\%$$

The PCI's are then compared across species to determine if the proposed methodology is sensitive to species differences through an analysis of variance methodology.

To augment the metric (10), we also computed for average similarity index (δ_s) per species:

$$(11) \dots \text{Average Similarity Index Per Species} = \frac{\sum_{i=1}^q \delta_{s_i}}{q}$$

4.0 Results and Discussions

Figure 2 shows the samples of mangrove leaves used in the study.

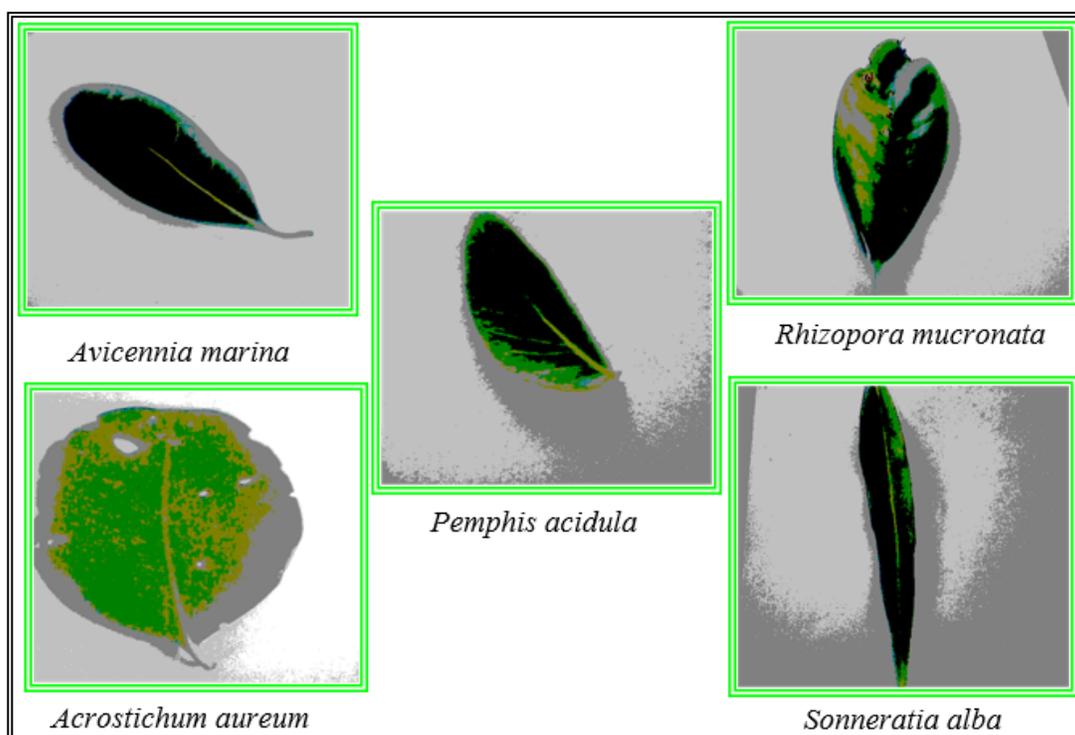


Figure 2. Mangrove leaves used in the study

Table 1. Summary of Empirical Fractal Dimensions of Mangrove Leaves

Trial	Fractal Dimensions of Mangrove Leaves				
	<i>Rhizophora mucronata</i>	<i>Acrostichum aureum</i>	<i>Avicennia marina</i>	<i>Sonneratia alba</i>	<i>Pemphis acidula</i>
1	1.9523	1.9722	1.9582	1.9496	1.9686
2	1.9536	1.9610	1.9525	1.9601	1.9584
3	1.9458	1.9579	1.9471	1.9454	1.9570
4	1.9596	1.9711	1.9600	1.9360	1.9639
5	1.9484	1.9692	1.9564	1.9465	1.9603
6	1.9447	1.9686	1.9466	1.9414	1.9535
7	1.9453	1.9628	1.9458	1.9321	1.9582
8	1.9478	1.9528	1.9528	1.9408	
9		1.9610	1.9610	1.9409	
10		1.9591	1.9591		

Table 2. Means and Standard Deviations of the Fractal Dimensions

Variable	N	Mean	Median	TrMean	StDev	SE Mean
<i>Rhizophora mucronata</i>	8	1.9497	1.9481	1.9497	0.0051	0.0018
<i>Acrostichum aureum</i>	10	1.9636	1.9619	1.9638	0.0064	0.0020
<i>Avicennia marina</i>	10	1.9540	1.9546	1.9541	0.0058	0.0018
<i>Sonneratia alba</i>	9	1.9436	1.9466	1.9466	0.0814	0.0027
<i>Pemphis acidula</i>	7	1.9600	1.9600	1.9600	0.0049	0.0019

Table 3. One-way ANOVA of the Mangrove Leaves

Analysis of Variance					
Source	DF	SS	MS	F	P
Factor	4	0.0022786	0.0005696	14.44	0.000
Error	39	0.0015388	0.0000395		
Total	43	0.0038174			

Table 4. Summary of t-values for comparing fractal dimensions

Pair	Mean difference	t-value	p-value
<i>Rhizophora</i> vs. <i>Acrostichum</i>	0.01388	5.10	0.000*
<i>Rhizophora</i> vs. <i>Avicennia</i>	0.00426	1.64	0.121
<i>Rhizophora</i> vs. <i>Sonneratia</i>	0.00304	1.87	0.087*
<i>Rhizophora</i> vs. <i>Pemphis</i>	0.01030	3.95	0.002*
<i>Acrostichum</i> vs. <i>Avicennia</i>	0.00962	3.51	0.002*
<i>Acrostichum</i> vs. <i>Sonneratia</i>	0.01993	5.85	0.000*
<i>Acrostichum</i> vs. <i>Pemphis</i>	0.00358	1.30	0.214
<i>Avicennia</i> vs. <i>Sonneratia</i>	0.01031	3.14	0.007*
<i>Avicennia</i> vs. <i>Pemphis</i>	0.00604	2.30	0.020*
<i>Sonneratia</i> vs. <i>Pemphis</i>	0.01634	4.96	0.000*
		Probability of Misclassification	0.0451

All the mangrove leaves display high fractal dimensions which demonstrate that mangrove leaves are indeed highly complex and rugged in shape and form. *Acrostichum aureum* and *Pemphis acidula* registered the highest fractal dimensions (beyond 1.96) while the most uniform in shape is *Sonneratia alba* (1.9436).

Comparison of the fractal dimensions between mangrove species revealed that the fractal dimensions of the five (5) mangrove leaves are significantly different ($f = 14.400$, $\rho = 0.00$). The high computed f-value could be attributed to the small standard errors of the mean (fractal dimension) computed for each species. This means that leaves belonging to the same species of mangrove have relatively the same fractal dimension in comparison to leaves belonging to different species. The similarity index computed for the fractal dimensions of the leaves exceeded 95%.

Further analysis, however, revealed that *Rhizophora mucronata* and *Avicennia marina* have statistically similar fractal

dimensions. This is true as well for *Acrostichum aureum* and *Pemphis acidula*. In other words, the proposed methodology using fractals in classifying the mangrove leaves across species would not be able to detect differences between the leaves of the aforementioned mangrove species. On the other hand, *Sonneratia alba* would be easily detected by the proposed methodology.

In the study of Mancuso (1997) and others (Ng et al., 2002), the number of leaves used for each species of plants was 20 or more. The larger sample size would have the effect of reducing the standard error of the mean of the fractal dimensions. It is therefore very likely that the findings of the present study would be different had more observations been taken. Due to time and resource limitations, however, we were constrained to reduce the observations to 10 or less.

With these limitations, it is possible to conduct further studies that would respond to scientific questions such as: Do water salinity and pH impact on the

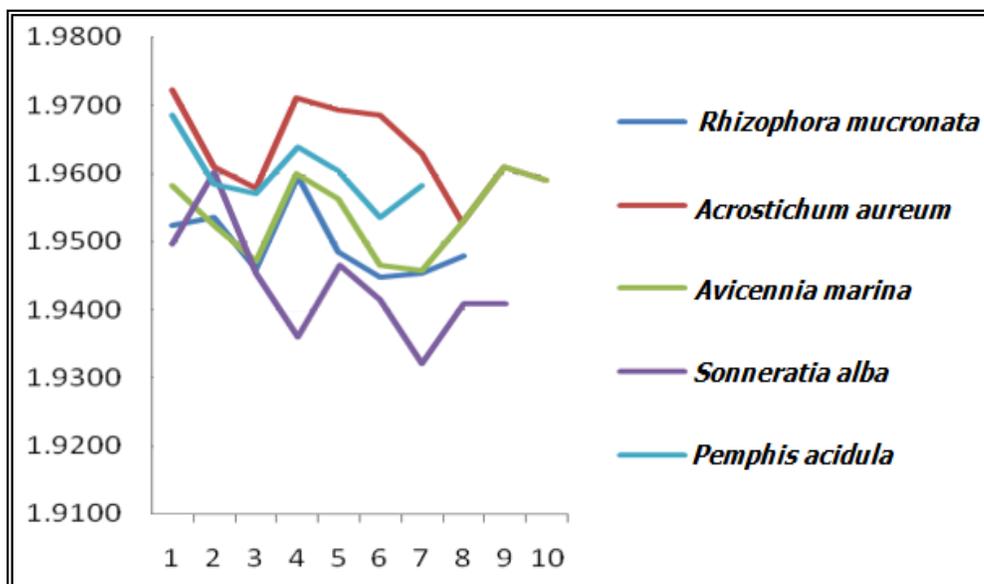


Figure 3. Graph of the fractal dimensions of selected mangrove leaves

fractal dimensions of the mangrove species? Can the carbon sequestration properties of mangrove species be deduced from knowledge of the fractal dimensions of their leaves? (see for example Boudon et al., (2004) who studied the carbon sequestration properties of other trees).

5.0 Conclusion

The methodology of using fractal dimensions of leaves of mangrove species to classify the mangroves themselves is a potentially powerful technique yielding a low misclassification probability (4.5% or less). Leaves belonging to the same mangrove species have equally high similarity index exceeding 95%. Across species, however, the fractal dimensions of mangrove leaves varied significantly beyond the 0.01 probability level.

Acknowledgement

The authors wish to acknowledge the technical assistance and guidance of Dr. Roberto N. Padua in the use of fractal statistics.

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